NPS55-84-007

NAVAL POSTGRADUATE SCHOOL

Monterey, California

RENORMALIZATION OF SEASONALS
IN THE ADDITIVE SEASONAL MODEL:
IS IT NECESSARY?

by

Ed McKenzie

February 1984

Approved for public release; distribution unlimited

Prepared for:

Naval Postgraduate School
Monterey, California 93943

NAVAL POSTGRADUATE SCHOOL
Monterey, California

This work was supported by the Naval Postgraduate School Foundation Research Program under contract with the National Research Council.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NPS55-84-007
2. GOVT ACCESSION NO.:
3. RECIPIENT'S CATALOG NUMBER:
4. TITLE (and Subtitle): RENORMALIZATION OF SEASONALS IN THE ADDITIVE SEASONAL MODEL: IS IT NECESSARY?
5. TYPE OF REPORT & PERIOD COVERED: Technical
6. PERFORMING ORG. REPORT NUMBER:
7. AUTHOR(s): Ed McKenzie
8. CONTRACT OR GRANT NUMBER(s):
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Postgraduate School, Monterey, CA 93943
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61152N; RR000-01-100, N0001483WR30104
11. CONTROLLING OFFICE NAME AND ADDRESS:
12. REPORT DATE: February 1984
13. NUMBER OF PAGES: 5
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office):
15. SECURITY CLASS. (of this report): Unclassified
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE:
16. DISTRIBUTION STATEMENT (of this Report): Approved for public release; distribution unlimited.
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report):
18. SUPPLEMENTARY NOTES:
19. KEY WORDS (Continue on reverse side if necessary and identify by block number):
Winters Additive Seasonal Model
Revision of Seasonal Factors
Renormalization of Seasonal Factors
20. ABSTRACT (Continue on reverse side if necessary and identify by block number):

It is always recommended that the initial estimates of the seasonal factors of the additive seasonal model sum to zero, i.e. are normalized. Once forecasting begins, however, the seasonal factors lose this property. It is often recommended that the factors are renormalized either seasonally or continuously, i.e. with every observation. We show that this latter procedure is never necessary since a simple change in the smoothing constants can achieve the same ends. A simpler and more efficient method of obtaining seasonally renormalized forecasts is also given.

DD FORM 1473, 1 JAN 73 — EDITION OF 1 NOV 65 IS OBSOLETE

S/N 0102-LF-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

S/N 0102- LF- 014- 6601

RENORMALIZATION OF SEASONALS IN THE ADDITIVE SEASONAL MODEL: IS IT NECESSARY?

Ed McKenzie
Department of Operations Research
Naval Postgraduate School
Monterey, California 93943

and

Department of Mathematics
University of Strathclyde
Glasgow, Scotland

February 1984

It is always recommended that the initial estimates of the seasonal factors of the additive seasonal model sum to zero, i.e. are normalized. Once forecasting begins, however, the seasonal factors lose this property. It is often recommended that the factors are renormalized either seasonally or continuously, i.e. with every observation. We show that this latter procedure is never necessary since a simple change in the smoothing constants can achieve the same ends. A simpler and more efficient method of obtaining seasonally renormalized forecasts is also given.

The additive seasonal model is given by

$$X_t = m + bt + s_k + a_t \tag{1}$$

where $t = rn+k$, and n is the length of the season. The values $\{s_k : k = 1,2,\ldots,n\}$ are additive seasonal factors and $\{a_t\}$ is a sequence of independent random variates of zero mean and variance σ^2 . In this model it is always assumed that the seasonal factors are normalized, i.e. sum to zero, $\sum_{k=1}^{n} s_k = 0$. This ensures a measure of independence between the level of the process (m) and the seasonal pattern. It helps to prevent changes in the seasonal factors being confused with changes in level and vice versa.

We are concerned here with the application of Winters' additive seasonal forecasting system to the model(1). This system is relatively simple to implement, intuitively appealing, and commonly used in practice. It is described in detail in most forecasting texts, e.g. Montgomery and Johnson [1976], Bowerman and O'Connell [1979] and Thomopoulos [1980]. When the system is implemented it is always recommended that the initial estimates of the seasonal factors sum to zero. Advice about what to do thereafter is less clear. As we shall see, the nature of the revision of the seasonal factors is such that they will no longer sum to zero after the first observation and revision. We can, however, renormalize the seasonal factors at any time. This is accomplished by obtaining the average of the set of estimated factors at time t and subtracting it from each estimated current factor. How and even whether such a procedure should be used is by no means clear. Some authors appear to recommend against its use, e.g. Bowerman and O'Connell [1979]; some regard it as an optional modification, e.g. Montgomery and Johnson [1976]; some recommend seasonal renormalization, i.e. once only per season, e.g. Chatfield [1978]; and some recommend continuous renormalization, i.e. after every revision of seasonal factors, e.g. Thomopoulos [1980].

The purpose of this paper is to derive a simpler and more efficient way of achieving seasonal and continuous renormalization. Indeed, we are able to show that continuous renormalization as such is an entirely unnecessary practice.

1. WINTERS' ADDITIVE SEASONAL SYSTEM

The form of the system corresponding to model (1) is given by

$$m_t = \alpha_0(X_t - S_{t-n}) + (1-\alpha_0)(m_{t-1} + b_{t-1}) \tag{2a}$$

$$bt = \alpha_1(m_t - m_{t-1}) + (1-\alpha_1)b_{t-1} \tag{2b}$$

$$S_t = \alpha_2(X_t - m_t) + (1-\alpha_2)S_{t-n} \tag{2c}$$

with the T-step ahead forecast

$$\hat{X}_t(T) = m_t + Tb_t + S_{t+k-n} \tag{3}$$

where $T = rn+k \quad (1 \leq k \leq n\ ,\ r \geq 0)$.

For easier use, equations (2a,b,c) are usually rewritten in the form

$$m_t = m_{t-1} + b_{t-1} + \alpha e_t \tag{4a}$$

$$b_t = b_{t-1} + \beta e_t \tag{4b}$$

$$S_t = S_{t-n} + \gamma e_t \tag{4c}$$

where $e_t = X_t - \hat{X}_{t-1}(1)$, and

$$\alpha = \alpha_0\ ,\ \beta = \alpha_0\alpha_1 \ \text{ and } \ \gamma = \alpha_2(1-\alpha_0)\ . \tag{5}$$

Equation (4c) gives the revision of the current seasonal factor at time t. All other seasonal factors, i.e. $S_{t-1}, S_{t-2},\ldots,S_{t-n+1}$, remain the same as at time $(t-1)$.

In what follows we shall be considering the effective revision of all n seasonal factors at every time t. To facilitate this we shall rewrite (4c) in an alternative form. We define the entire set of seasonal factors at time t as $\{S_t^k : k = 1,2,\ldots,n\}$. In accordance with the previous ideas, S_t^k is the seasonal factor corresponding to $rn+k$ periods ahead at time t. Thus, S_{t+k-n} becomes S_t^k. Further, (4c) is replaced by

$$\begin{aligned} S_t^k &= S_{t-1}^{k+1}, \quad k = 1,2,\ldots,n-1, \\ S_t^n &= S_{t-1}^1 + \gamma e_t . \end{aligned} \tag{4d}$$

2. RENORMALIZATION OF THE SEASONAL FACTORS

Suppose we are using (4a,b,d) to generate forecasts and wish to renormalize the seasonal factors at time t. After the revision of (4d) the seasonal factors are each reduced by the average of the set. This can be achieved in a single step by replacing (4d) and the subsequent reduction by the following revision equations:

$$\begin{aligned} S_t^k &= S_{t-1}^{k+1} - \gamma e_t/n - \bar{S}_{t-1}, \quad k = 1,2,\ldots,n-1, \\ S_t^n &= S_{t-1}^1 + (1-\gamma/n)e_t - \bar{S}_{t-1}, \end{aligned} \tag{6}$$

where $\bar{S}_{t-1} = \sum_{k=1}^{n} S_{t-1}^k/n$.

We consider now two particular forms of application.

(i) Continuous renormalization.

In this case the seasonal factors are renormalized at every time t, and so $\bar{S}_t = 0$, for all t. Thus, the season factors obtained from (6) are the same as those given by (4d) except that each is reduced by $\gamma e_t/n$. Note, however, that every forecast $\hat{X}_t(T)$ contains m_t and S_t^k only in an

expression of the form $(m_t + S_t^k)$. Thus, the same effect as renormalization can be achieved by revising S_t^k with (4d) and reducing m_t by $\gamma e_t/n$. Equation (4a) then becomes

$$m_t = m_{t-1} + b_{t-1} + (\alpha-\gamma/n)e_t \ . \tag{7}$$

Using (7) and (4b,d) yields exactly the same forecasts as using (4a,b,d) and renormalizing the seasonals.

Notice that we can achieve the effect of continuous renormalization using (4a,b,c) if we simply change α to $(\alpha-\gamma/n)$. However, if we wish to use the original form (2a,b,c) to forecast, the changes in the smoothing constants are a little more complex but may be obtained from (5). In fact, $\alpha_0,\alpha_1,\alpha_2$ must be replaced by β_0,β_1,β_2 respectively, where

$$\beta_0 = \alpha_0 - \alpha_2(1-\alpha_0)/n \ , \quad \beta_1 = \alpha_0\alpha_1/\beta_0 \ , \quad \beta_2 = \alpha_2/(1+\alpha_2/n) \tag{8}$$

It is important to emphasize the interpretation of the result above. If we use equations (2a,b,c) with smoothing constants β_0,β_1,β_2 , or else equations (4a,b,c) with constants $(\alpha-\gamma/n)$, β, γ , then we generate exactly the same forecasts, for all lead times, as would occur if we used $\alpha_0,\alpha_1,\alpha_2$, in (2a,b,c) or α,β,γ in (4a,b,c) and renormalized the seasonal factors with every observation. We do not obtain the same values of m_t and S_t^k , $k = 1,2,\ldots,n$. We do obtain the same values of $(m_t + S_t^k)$, $k = 1,2,\ldots,n$. With this alternative procedure the seasonal factors we obtain do not sum to zero. If we wish to obtain a set which do so at any time we simply reduce them by their average value. However, if we wish only forecasts the alternative procedure is clearly a simpler and computationally more efficient way to obtain them.

An obvious consequence of the result obtained here is that if we wish to forecast using (2a,b,c) or the more general (4a,b,c) then continuous

renormalization is a redundant procedure. We can obtain the same forecasts by suitably altering the smoothing constants as discussed above.

(ii) Seasonal renormalization.

In this case, we suppose $\sum_{k=1}^{n} S_t^k = 0$ and we forecast and revise the model parameters using (2a,b,c) or (4a,b,c) until time (t+n) when the seasonal factors are renormalized once again. The procedure is repeated every season, i.e. every n^{th} revision.

It is easily shown using (4d) that

$$S_{t+n}^k = S_t^k + \gamma e_{t+k} , \quad k = 1,2,\ldots,n .$$

If we renormalize at (t+n) as before by reducing each factor by the average, we find S_{t+n}^k is reduced by $\gamma\bar{e}_t$, where $\bar{e}_t = \sum_{k=1}^{n} e_{t+k}/n$, the average of the season's one-step ahead errors. Again, the same forecasts may be achieved more easily by revising S_{t+n}^k in the usual way, i.e. (4d), and reducing m_t by $\gamma\bar{e}_t$. This procedure cannot be easily incorporated into the structure of the revision equations since $\bar{e}_t$ involves the last n errors and not simply the latest. Nevertheless, the procedure is a simpler and more efficient way of obtaining the same forecasts as seasonal renormalization would yield. Thus, we use (2a,b,c) or (4a,b,c) during the season, i.e. for all n revisions, and, after the n^{th} revision, we reduce m_{t+n} by $\gamma\bar{e}_t$. We can calculate $\bar{e}_t$ recursively and only one extra value need be stored.

ACKNOWLEDGMENT

I gratefully acknowledge the support of a National Research Council Associateship at the Naval Postgraduate School in Monterey, California, where this work was carried out.

REFERENCES

Bowerman, B. L. and R. T. O'Connell. 1976. Time Series and Forecasting: an applied approach. Duxbury Press, Massachusetts.

Chatfield, C. 1978. The Holt-Winters Forecasting Procedure. Appl. Statistcs. 27, 264-279.

Montgomery, D. C. and L. A. Johnson. 1976. Forecasting and Time Series Analysis. McGraw-Hill, New York.

Thomopoulos, N. T. 1980. Applied Forecasting Methods. Prentice-Hall, New Jersey.

DISTRIBUTION LIST

	NO. OF COPIES
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Library Code 0142 Naval Postgraduate School Monterey, CA 93943	2
Research Administration Code 012A Naval Postgraduate School Monterey, CA 93943	1
Library Code 55 Naval Postgraduate School Monterey, CA 93943	1
Professor Ed McKenzie Code 55 Naval Postgraduate School Monterey, CA 93943	60
Professor P. A. W. Lewis Code 55Lw Naval Postgraduate School Monterey, CA 93943	10